Erratum/Addendum for *Topics in Spectral Geometry*, preliminary online version dated May 29, 2023

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☞ denotes recently added items to Erratum/Addendum

$\omega\chi$ ERRATUM

☞ **p. 13, line 3:** replace "rises" by "raises"

☞ **p. 36, line 4 of the paragraph directly above the heading of §2.1.2:** replace "compact" by "closed"

☞ **p.40, formula (2.1.6):** in the second integral, replace "dσ" by "ds"

☞ **p. 39, line following (2.1.5):** remove the word from "that for ~~for~~"

☞ **p. 76, line -3:** remove the word from "real ~~domain~~ analyticity"

☞ **p. 53, formula (2.2.3):** in the left-hand side, add a missing f to get $\|\partial_{te_k} f\|$

☞ **p. 81, first line of Exercise 3.2.14(iii):** replace "disjoined" by "disjoint"

☞ **p. 93, lines -5 and -4:** in both places, replace "$N^{\mathrm{D}}(\lambda)$" by "$\mathcal{N}^{\mathrm{D}}(\lambda)$"

☞ **pp. 116 and 117:** as stated, Theorem 4.1.11 can be found in [Kin21, Corollary 4.31]. However, in the proof of Theorem 4.1.6 we in fact use a version from [HeiKilMar93, Theorem 4.5], which is formulated slightly differently:

Theorem 4.1.11′. *Let Ω be an open subset of $\mathbb{R}^d$. Then the function $u \in H^1(\Omega)$ belongs to $H^1_0(\Omega)$ if and only if there exists a quasi-continuous function v on $\mathbb{R}^d$ such that $v(x) = 0$ quasi-everywhere outside Ω and $v(x) = u(x)$ almost everywhere in Ω.*

In the proof of Theorem 4.1.6 (which remains unchanged), we have $u|_{\Omega_1} \in H^1(\Omega_1)$, and we construct a quasi-continuous w such that $w(x) = 0$ quasi-everywhere outside Ω_1 and $w(x) = u(x)$ almost everywhere in Ω_1. Thus $u|_{\Omega_1} \in H^1_0(\Omega_1)$ by Theorem 4.1.11′ given above.

We thank R. L. Frank for pointing this out.

☞ **p. 141, second line of the second paragraph:** replace "has led" by 'have led"

☞ **p. 153, first line after the statement of Theorem 5.1.4:** replace "have measure" by "has measure"

☞ **p. 238, two lines above formula (7.1.15):** replace "$L(\Omega_k)$" by "$L(\partial\Omega_k)$"

☞ **p. 238, formula (7.1.15):** replace "$L(\Omega)$" by "$L(\partial\Omega)$"

☞ **p. 238, line following formula (7.1.15):** add the words to get "on any surface of genus zero with boundary"

☞ **p. 256, line -1:** replace "Exercise 7.3.6(iv)" by "Exercise 7.3.6(iii)"

☞ **p. 260, third line above Theorem 7.3.8:** replace "(7.3.8), and (7.3.8) imply" by "(7.3.8), and (7.3.9) imply"

☞ **p. 266, fourth line of Exercise 7.3.15:** replace "Figure 7.2" by "Figure 7.3"

☞ **p. 266, Exercise 7.4.3** replace the first displayed formula by

$$\begin{cases} \dfrac{\sqrt{-\Lambda}\,I_0'(\sqrt{-\Lambda})}{I_0(\sqrt{-\Lambda})}, & \text{if } \Lambda < 0, \\ 0, & \text{if } \Lambda = 0, \\ \dfrac{\sqrt{\Lambda}\,J_0'(\sqrt{\Lambda})}{J_0(\sqrt{\Lambda})}, & \text{if } \Lambda > 0, \end{cases}$$

and replace the second displayed formula by

$$\begin{cases} \dfrac{\sqrt{-\Lambda}\,I_m'(\sqrt{-\Lambda})}{I_m(\sqrt{-\Lambda})}, & \text{if } \Lambda < 0, \\ m, & \text{if } \Lambda = 0, \\ \dfrac{\sqrt{\Lambda}\,J_m'(\sqrt{\Lambda})}{J_m(\sqrt{\Lambda})}, & \text{if } \Lambda > 0, \end{cases} \qquad m \in \mathbb{N},$$

☞ **p. 277, first line of Remark 7.4.14:** replace "Theorem 7.2.11" by "Theorem 7.4.11"

Σ ADDENDUM

☞ **p. 28, two lines above Remark 1.2.14:** since an issue with the preprint [BouWat17] became known in July 2023, the best existing upper bound for $R(\lambda)$ has exponent $\frac{131}{416} + \varepsilon$ [M. N. Huxley, *Exponential sums and lattice points III*, Proc. London Math. Soc. (3) **87** (2003), 591–609. doi: 10.1112/S0024611503014485]

☞ **p. 68, above Exercise 3.1.6:** on reflection, the phrase "Proposition 3.1.3 follows immediately" is somewhat misleading. Therefore, for methodological purposes, we extend the argument outlining the proof of Proposition 3.1.3. Let $\mathscr{E}_k := \mathrm{Span}\{u_k, u_{k+1}, \dots\}$. Then $R[u] \geq \lambda_k$ for any $u \in \mathscr{E}_k \setminus \{0\}$. If we now consider an arbitrary $\mathscr{L} \subset \mathrm{Dom}(\mathscr{Q})$ with $\dim \mathscr{L} = k$, then there exists $u \in \mathscr{E}_k \cap \mathscr{L} \setminus \{0\}$ (since the codimension of $\mathscr{E}_k$ is $k-1$), and therefore

$$\max_{u \in \mathscr{L} \setminus \{0\}} R[u] \geq \lambda_k.$$

On the other hand, the equality is attained if we take $\mathscr{L} = \mathrm{Span}\{u_1, \dots, u_k\}$.

☞ **p. 317, reference [ColGGS22]:** add bibliographical data, Rev. Mat. Complut. (2023). doi: 10.1007/s13163-023-00480-3

☞ **p. 319, reference [FilLPS23]:** add full bibliographical data, Invent. Math. **234** (2023), 129–169. doi: 10.1007/s00222-023-01198-1

☞ **p. 324, reference [KarLagPol22]:** add full bibliographical data, Arch. Rational Mech. Anal. **47** (2023), article no. 77. doi: 10.1007/s00205-023-01912-6

☞ **p. 331, reference [Ros22b]:** add full bibliographical data, Math. Z. **305** (2023), article 62. doi: 10.1007/s00209-023-03382-8